

1.

Resuelve las ecuaciones siguientes:

a)  $x^4 - x^2 - 12 = 0$

b)  $x^4 - 8x^2 - 9 = 0$

a)  $x^2 = \frac{1 \pm \sqrt{1+48}}{2} = \frac{1 \pm 7}{2} \begin{cases} 4 \rightarrow x = \pm 2 \\ -3 \rightarrow \text{(no vale)} \end{cases} \quad 2 \text{ y } -2$

b)  $x^2 = \frac{8 \pm \sqrt{64+36}}{2} = \frac{8 \pm 10}{2} \begin{cases} 9 \rightarrow x = \pm 3 \\ -1 \rightarrow \text{(no vale)} \end{cases} \quad 3 \text{ y } -3$

2.

Resuelve:

a)  $x^4 + 10x^2 + 9 = 0$

b)  $x^4 - x^2 - 2 = 0$

a)  $x^2 = \frac{-10 \pm \sqrt{100-36}}{2} = \frac{-10 \pm 8}{2} \begin{cases} -1 \rightarrow \text{(no vale)} \\ -9 \rightarrow \text{(no vale)} \end{cases}$

No tiene solución.

b)  $x^4 - x^2 - 2 = 0$

$$x^2 = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2} \begin{cases} x^2 = -1 \rightarrow \text{No vale} \\ x^2 = 2 \rightarrow x = \pm \sqrt{2} \end{cases}$$

Hay dos soluciones:  $x_1 = -\sqrt{2}$ ;  $x_2 = \sqrt{2}$ 

3.

Resuelve:

a)  $-\sqrt{2x-3} + 1 = x$

b)  $\sqrt{2x-3} - \sqrt{x+7} = 4$

c)  $2 + \sqrt{x} = x$

d)  $2 - \sqrt{x} = x$

e)  $\sqrt{3x+3} - 1 = \sqrt{8-2x}$

a)  $1 - x = \sqrt{2x-3}$

$1 + x^2 - 2x = 2x - 3; \quad x^2 - 4x + 4 = 0; \quad x = 2 \text{ (no vale)}$

No tiene solución.

b)  $2x - 3 = 16 + x + 7 + 8\sqrt{x+7}$

$x - 26 = 8\sqrt{x+7}$

$x^2 + 676 - 52x = 64(x+7)$

$x^2 + 676 - 52x = 64x + 448$

$x^2 - 116x + 228 = 0; \quad x = \frac{116 \pm 112}{2} \begin{cases} 114 \\ 2 \rightarrow \text{(no vale)} \end{cases}$

$x = 114$

$$\begin{aligned} \text{c) } \sqrt{x} &= x - 2; \quad x = x^2 + 4 - 4x; \quad 0 = x^2 - 5x + 4 \\ x &= \frac{5 \pm \sqrt{25 - 16}}{2} = \frac{5 \pm 3}{2} \begin{cases} 4 \\ 1 \end{cases} \rightarrow (\text{no vale}) \\ x &= 4 \end{aligned}$$

$$\begin{aligned} \text{d) } 2 - x &= \sqrt{x}; \quad 4 + x^2 - 4x = x; \quad x^2 - 5x + 4 = 0 \\ x &= \begin{cases} 4 \\ 1 \end{cases} \rightarrow (\text{no vale}) \\ x &= 1 \end{aligned}$$

$$\begin{aligned} \text{e) } \sqrt{3x + 3} - 1 &= \sqrt{8 - 2x} \\ 3x + 3 &= 1 + 8 - 2x + 2\sqrt{8 - 2x} \\ 5x - 6 &= 2\sqrt{8 - 2x} \\ 25x^2 + 36 - 60x &= 4(8 - 2x) \\ 25x^2 - 52x + 4 &= 0 \\ x &= \frac{52 \pm 48}{50} \begin{cases} x = 2 \\ x = 0,08 \end{cases} \rightarrow \text{no vale} \\ \text{Así, } x &= 2. \end{aligned}$$

#### 4.

Resuelve las siguientes ecuaciones:

$$\text{a) } \frac{1}{x} + \frac{1}{x+3} = \frac{3}{10} \qquad \text{b) } \frac{4}{x} + \frac{2(x+1)}{3(x-2)} = 4 \qquad \text{c) } \frac{1}{x} + \frac{1}{x^2} = \frac{3}{4}$$

$$\begin{aligned} \text{a) } 10(x+3) + 10x &= 3x(x+3) \\ 10x + 30 + 10x &= 3x^2 + 9x \\ 0 &= 3x^2 - 11x - 30 \\ x &= \frac{11 \pm 21,93}{6} = \begin{cases} 5,489 \\ -1,822 \end{cases} \\ x_1 &= 5,489; \quad x_2 = -1,822 \end{aligned}$$

$$\begin{aligned} \text{b) } 12(x-2) + 2x(x+1) &= 12x(x-2) \\ 12x - 24 + 2x^2 + 2x &= 12x^2 - 24x \\ 0 &= 10x^2 - 38x + 24 \\ 0 &= 5x^2 - 19x + 12; \quad x = \frac{19 \pm 11}{10} = \begin{cases} 3 \\ 4/5 \end{cases} \\ x_1 &= 3; \quad x_2 = \frac{4}{5} \end{aligned}$$

$$\begin{aligned} \text{c) } 4x + 4 &= 3x^2; \quad 0 = 3x^2 - 4x - 4 \\ x &= \frac{4 \pm 8}{6} = \begin{cases} 2 \\ -2/3 \end{cases} \\ x_1 &= 2; \quad x_2 = \frac{-2}{3} \end{aligned}$$

5.

Resuelve las siguientes ecuaciones:

a)  $2^{3x} = 0,5^{3x+2}$

b)  $3^{4-x^2} = \frac{1}{9}$

c)  $\frac{4^{x-1}}{2^{x+2}} = 186$

d)  $7^{x+2} = 5\,764\,801$

a)  $2^{3x} = 2^{-3x-2}$ ;  $3x = -3x - 2$ ;  $6x = -2$ ;  $x = \frac{-1}{3}$

b)  $3^{4-x^2} = 3^{-2}$ ;  $4 - x^2 = -2$ ;  $x^2 = 6$ ;  $x = \pm\sqrt{6}$

$x_1 = \sqrt{6}$ ;  $x_2 = -\sqrt{6}$

c)  $\frac{2^{2x-2}}{2^{x+2}} = 186$ ;  $2^{2x-2-x-2} = 186$ ;  $2^{x-4} = 186$

$\log 2^{x-4} = \log 186$ ;  $(x-4) \log 2 = \log 186$

$x = 4 + \frac{\log 186}{\log 2} = 11,54$

d)  $7^{x+2} = 7^8$ ;  $x = 6$

6.

Resuelve:

a)  $3^x + 3^{x+2} = 30$

b)  $5^{x+1} + 5^x + 5^{x-1} = \frac{31}{5}$

c)  $2 \log x - \log(x+6) = 3 \log 2$

d)  $4 \log_2(x^2+1) = \log_2 625$

a)  $3^x + 3^x \cdot 9 = 30$

$3^x(10) = 30$ ;  $3^x = 3$ ;  $x = 1$

b)  $5 \cdot 5^x + 5^x + \frac{5^x}{5} = \frac{31}{5}$

$5^x \cdot \frac{31}{5} = \frac{31}{5}$ ;  $x = 0$

c)  $\log \frac{x^2}{x+6} = \log 8$

$x^2 = 8x + 48$ ;  $x^2 - 8x - 48 = 0$ ;  $x = \frac{8 \pm 16}{2} = \begin{cases} 12 \\ -4 \end{cases}$  (no vale)

$x = 12$

d)  $\log_2(x^2+1)^4 = \log_2 5^4$ ;  $x^2+1 = 5$ ;  $x^2 = 4$ ;  $x = \pm 2$

$x_1 = 2$ ;  $x_2 = -2$

7.

Resuelve estos sistemas de ecuaciones:

$$\text{a) } \begin{cases} 2x - y - 1 = 0 \\ x^2 - 7 = y + 2 \end{cases} \quad \text{b) } \begin{cases} \frac{1}{x} + \frac{1}{y} = 1 - \frac{1}{xy} \\ xy = 6 \end{cases} \quad \text{c) } \begin{cases} x = 2y + 1 \\ \sqrt{x+y} - \sqrt{x-y} = 2 \end{cases}$$

$$\text{a) } \begin{cases} y = 2x - 1 \\ y = x^2 - 9 \end{cases}$$

$$x^2 - 9 = 2x - 1; \quad x^2 - 2x - 8 = 0$$

$$x = \frac{2 \pm \sqrt{4 + 32}}{2} = \frac{2 \pm 6}{2} = \begin{cases} 4 \\ -2 \end{cases}$$

$$x_1 = 4; \quad y_1 = 7$$

$$x_2 = -2; \quad y_2 = -5$$

$$\text{b) } \begin{cases} y + x = xy - 1 \\ xy = 6 \end{cases}$$

$$y = 5 - x$$

$$x(5 - x) = 6; \quad 5x - x^2 = 6; \quad x^2 - 5x + 6 = 0 \begin{cases} x = 2 \\ x = 3 \end{cases}$$

$$x_1 = 2; \quad y_1 = 3$$

$$x_2 = 3; \quad y_2 = 2$$

$$\text{c) } x = 2y + 1$$

$$\sqrt{3y+1} - \sqrt{y+1} = 2; \quad \sqrt{3y+1} = 2 + \sqrt{y+1}$$

$$3y+1 = 4 + y+1 + 4\sqrt{y+1}; \quad 2y-4 = 4\sqrt{y+1}; \quad y-2 = 2\sqrt{y+1}$$

$$y^2 + 4 - 4y = 4y + 4; \quad y^2 - 8y = 0$$

$$y = 8 \rightarrow x = 17$$

$$y = 0 \text{ (no vale)}$$

$$x = 17; \quad y = 8$$

8.

$$\text{a) } \begin{cases} x^2 + xy + y^2 = 21 \\ x + y = 1 \end{cases} \quad \text{b) } \begin{cases} x - y = 27 \\ \log x - 1 = \log y \end{cases} \quad \text{c) } \begin{cases} \log(x^2 + y) - \log(x - 2y) = 1 \\ 5^{x+1} = 25^{y+1} \end{cases}$$

$$\text{a) } y = 1 - x; \quad x^2 + x(1-x) + (1-x)^2 = 21$$

$$x^2 + x - x^2 + 1 + x^2 - 2x = 21; \quad x^2 - x - 20 = 0$$

$$x = \frac{1 \pm \sqrt{1 + 80}}{2} = \frac{1 \pm 9}{2} = \begin{cases} 5 \rightarrow y = -4 \\ -4 \rightarrow y = 5 \end{cases}$$

$$x_1 = -4; \quad y_1 = 5$$

$$x_2 = 5; \quad y_2 = -4$$

$$\text{b) } \begin{cases} x = 27 + y \\ \log \frac{x}{y} = 1 \end{cases}$$

$$10y = 27 + y; \quad 9y = 27; \quad y = 3$$

$$\frac{x}{y} = 10; \quad x = 10y; \quad x = 30$$

$$x = 30; \quad y = 3$$

$$c) \left. \begin{aligned} \log \frac{x^2 + y}{x - 2y} &= 1 \\ 5^{x+1} &= 5^{2y+2} \end{aligned} \right\}$$

$$\left. \begin{aligned} x^2 + y &= 10x - 20y \\ x + 1 &= 2y + 2 \end{aligned} \right\}$$

$$x = 2y + 1$$

$$4y^2 + 1 + 4y + y = 20y + 10 - 20y$$

$$4y^2 + 5y - 9 = 0$$

$$y = \frac{-5 \pm \sqrt{25 + 144}}{8} = \frac{-5 \pm 13}{8} = \begin{cases} -9/4 \rightarrow x = -7/2 \\ 1 \rightarrow x = 3 \end{cases}$$

$$x_1 = 3; y_1 = 1$$

$$x_2 = \frac{-7}{2}; y_2 = \frac{-9}{4}$$

9.

Resuelve, tomando logaritmos, estas ecuaciones:

a)  $\frac{1}{e^x} = 27$

b)  $e^{x-9} = \sqrt{73}$

c)  $2^x \cdot 3^x = 81$

d)  $\frac{2^x}{3^{x+1}} = 1$

a)  $\frac{1}{e^x} = 27 \rightarrow \frac{1}{27} = e^x \rightarrow \ln \frac{1}{27} = \ln e^x$

$$x = \ln \frac{1}{27} = \ln 1 - \ln 27 = 0 - \ln 27 \rightarrow x \approx -3,296$$

b)  $e^{x-9} = \sqrt{73} \rightarrow \ln e^{x-9} = \ln \sqrt{73}$

$$x - 9 = \frac{1}{2} \ln 73 \rightarrow x = 9 + \frac{\ln 73}{2} \rightarrow x \approx 11,145$$

c)  $6^x = 81; x \log 6 = \log 81$

$$x = \frac{\log 81}{\log 6} \approx 2,453$$

d)  $\frac{2^x}{3^x \cdot 3} = 1; \left(\frac{2}{3}\right)^x = 3; x \log \frac{2}{3} = \log 3$

$$x = \frac{\log 3}{\log 2 - \log 3} \approx -2,710$$

10.

Resuelve las siguientes ecuaciones mediante un cambio de variable:

a)  $2^x + 2^{1-x} = 3$

b)  $2^{x+1} + 2^{x-1} = \frac{5}{2}$

c)  $8^{1+x} + 2^{3x-1} = \frac{17}{16}$

d)  $2^{2x} - 5 \cdot 2^x + 4 = 0$

e)  $9^x - 3^x - 6 = 0$

f)  $7^{1+2x} - 50 \cdot 7^x + 7 = 0$

a)  $2^x + \frac{2}{2^x} = 3$

$$z = 2^x \rightarrow z + \frac{2}{z} = 3; \quad z^2 + 2 = 3z$$

$$z^2 - 3z + 2 = 0; \quad z = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2} = \begin{matrix} 2 \\ 1 \end{matrix}$$

$$2^x = 2 \rightarrow x_1 = 1; \quad 2^x = 1 \rightarrow x_2 = 0$$

b)  $2 \cdot 2^x + \frac{2^x}{2} = \frac{5}{2}; \quad 4 \cdot 2^x + 2^x = 5; \quad 2^x = 1$

$$x = 0$$

c)  $2^{3+3x} + 2^{3x-1} = \frac{17}{16}$

$$8 \cdot (2^x)^3 + \frac{(2^x)^3}{2} = \frac{17}{16} \rightarrow 2^x = z \rightarrow 128z^3 + 8z^3 = 17$$

$$(128 + 8)(z)^3 = 17; \quad (z)^3 = \frac{17}{136} = \frac{1}{8} \rightarrow z = \sqrt[3]{\frac{1}{8}} = \frac{1}{2} \rightarrow 2^x = \frac{1}{2}$$

$$x = -1$$

d)  $(2^x)^2 - 5 \cdot 2^x + 4 = 0$

$$2^x = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} = \begin{matrix} 4 \\ 1 \end{matrix}$$

$$x_1 = 0; \quad x_2 = 2$$

e)  $(3^x)^2 - 3^x - 6 = 0; \quad 3^x = \frac{1 \pm \sqrt{1+24}}{2} = \frac{1 \pm 5}{2} = \begin{matrix} 3 \\ -2 \end{matrix}$  (no vale)

$$x = 1$$

f)  $7 \cdot (7^x)^2 - 50 \cdot 7^x + 7 = 0; \quad 7^x = \frac{50 \pm 48}{14} = \begin{matrix} 7 \\ 1/7 \end{matrix}$

$$x_1 = -1; \quad x_2 = 1$$

11.

Resuelve las ecuaciones:

a)  $\log(x^2 + 1) - \log(x^2 - 1) = \log \frac{13}{12}$

b)  $\ln(x - 3) + \ln(x + 1) = \ln 3 + \ln(x - 1)$

c)  $2\ln(x - 3) = \ln x - \ln 4$

d)  $\log(x + 3) - \log(x - 6) = 1$

a)  $\log \frac{x^2 + 1}{x^2 - 1} = \log \frac{13}{12}$

$$12x^2 + 12 = 13x^2 - 13; \quad 25 = x^2$$

$$x_1 = -5; \quad x_2 = 5$$

b)  $\ln(x^2 - 2x - 3) = \ln(3x - 3)$

$$x^2 - 2x - 3 = 3x - 3; \quad x^2 - 5x = 0$$

$$x = 5 \quad (x = 0 \text{ no vale})$$

c)  $\ln(x - 3)^2 = \ln \frac{x}{4}$

$$x^2 + 9 - 6x = \frac{x}{4}$$

$$4x^2 + 36 - 24x = x; \quad 4x^2 - 25x + 36 = 0$$

$$x = \frac{25 \pm 7}{8} = \begin{cases} 4 \\ 9/4 \end{cases} \quad (9/4 \text{ no vale})$$

$$x = 4$$

d)  $\log \frac{x + 3}{x - 6} = 1$

$$x + 3 = 10x - 60; \quad 63 = 9x$$

$$x = 7$$

12.

Resuelve:

a)  $\begin{cases} x \cdot y = 15 \\ \frac{x}{y} = \frac{5}{3} \end{cases}$

b)  $\begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{5}{6} \\ 2x + 3y = 2 \end{cases}$

c)  $\begin{cases} x^2 + y^2 - 5x - 5y + 10 = 0 \\ x^2 - y^2 - 5x + 5y + 2 = 0 \end{cases}$

d)  $\begin{cases} (x + y)(x - y) = 7 \\ 3x - 4y = 0 \end{cases}$

$$\text{a) } x = \frac{5y}{3}$$

$$\frac{5y^2}{3} = 15; y^2 = 9 \begin{cases} y = 3 \rightarrow x = 5 \\ y = -3 \rightarrow x = -5 \end{cases}$$

$$x_1 = 5, y_1 = 3; x_2 = -5, y_2 = -3$$

$$\text{b) } \left. \begin{array}{l} 6y + 6x = 5xy \\ y = \frac{2-2x}{3} \end{array} \right\} \begin{array}{l} 4 - 4x + 6x = \frac{5x(2-2x)}{3} \\ 6x + 12 = 10x - 10x^2 \\ 10x^2 - 4x + 12 = 0 \\ 5x^2 - 2x + 6 = 0 \end{array}$$

No tiene solución.

$$\text{c) } 2x^2 - 10x + 12 = 0; x^2 - 5x + 6 = 0$$

$$x = \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm 1}{2} = \begin{cases} 3 \\ 2 \end{cases}$$

$$x^2 + y^2 - 5x - 5y + 10 = 0$$

$$-x^2 + y^2 + 5x - 5y - 2 = 0$$

$$\hline 2y^2 - 10y + 8 = 0$$

$$y^2 - 5y + 4 = 0$$

$$y = \frac{5 \pm \sqrt{25 - 16}}{2} = \frac{5 \pm 3}{2} = \begin{cases} 4 \\ 1 \end{cases}$$

$$x_1 = 3, y_1 = 4; x_2 = 3, y_2 = 1; x_3 = 2, y_3 = 4; x_4 = 2, y_4 = 1$$

$$\text{d) } x = \frac{4y}{3}$$

$$\frac{7y}{3} \cdot \frac{y}{3} = 7$$

$$y^2 = 9; y = \pm 3$$

$$x_1 = 4, y_1 = 3; x_2 = -4, y_2 = -3$$