Ejercicios de Matemáticas I 1º Bachillerato **HOJA 6**

1.

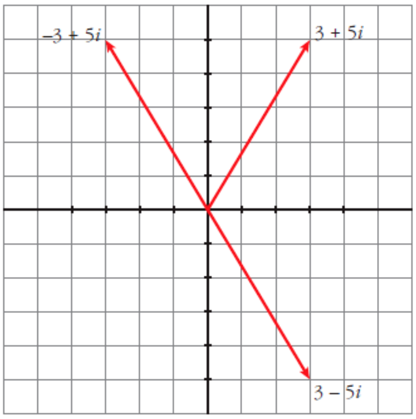
Representa gráficamente el opuesto y el conjugado de:

a) $3 - 5i$ **b)** $5 + 2i$ **c)** $-1 - 2i$ **d)** $-2 + 3i$

e) 5 **f)** 0 **g)** $2i$ **h)** $-5i$

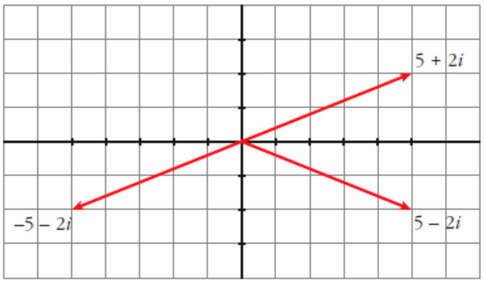
a) Opuesto: $-3 + 5i$

Conjugado: $3 + 5i$



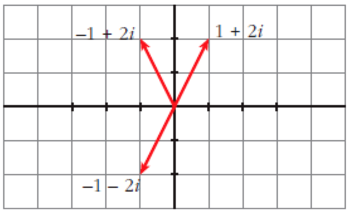
b) Opuesto: $-5 - 2i$

Conjugado: $5 - 2i$



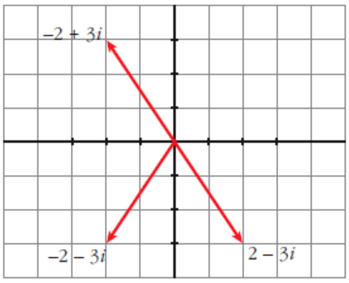
c) Opuesto: $1 + 2i$

Conjugado: $-1 + 2i$



d) Opuesto: $2 - 3i$

Conjugado: $-2 - 3i$

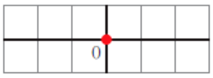


e) Opuesto: -5

Conjugado: 5

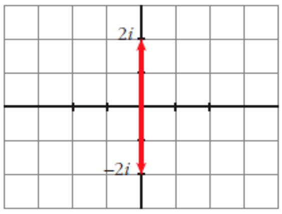
f) Opuesto: 0

Conjugado: 0



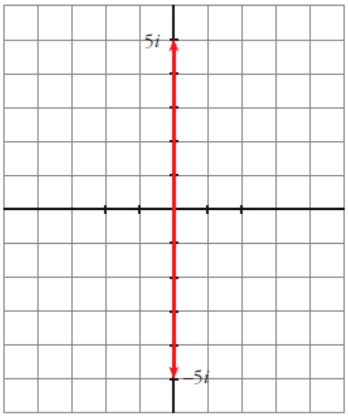
g) Opuesto: $-2i$

Conjugado: $-2i$



h) Opuesto: $5i$

Conjugado: $5i$



2.

Efectúa las siguientes operaciones y simplifica el resultado:

a) $(6 - 5i) + (2 - i) - 2(-5 + 6i)$

b) $(2 - 3i) - (5 + 4i) + \frac{1}{2}(6 - 4i)$

c) $(3 + 2i)(4 - 2i)$

d) $(2 + 3i)(5 - 6i)$

e) $(-i + 1)(3 - 2i)(1 + 3i)$

f) $\frac{2 + 4i}{4 - 2i}$ **g)** $\frac{1 - 4i}{3 + i}$ **h)** $\frac{4 + 4i}{-3 + 5i}$

i) $\frac{5 + i}{-2 - i}$ **j)** $\frac{1 + 5i}{3 + 4i}$ **k)** $\frac{4 - 2i}{i}$

l) $6 - 3\left(5 + \frac{2}{5}i\right)$ **m)** $\frac{(-3i)^2(1 - 2i)}{2 + 2i}$

a) $(6 - 5i) + (2 - i) - 2(-5 + 6i) = 6 - 5i + 2 - i + 10 - 12i = 18 - 18i$

b) $(2 - 3i) - (5 + 4i) + \frac{1}{2}(6 - 4i) = 2 - 3i - 5 - 4i + 3 - 2i = -9i$

c) $(3 + 2i)(4 - 2i) = 12 - 6i + 8i - 4i^2 = 12 + 2i + 4 = 16 + 2i$

d) $(2 + 3i)(5 - 6i) = 10 - 12i + 15i - 18i^2 = 10 + 3i + 18 = 28 + 3i$

e) $(-i + 1)(3 - 2i)(1 + 3i) = (-3i + 2i^2 + 3 - 2i)(1 + 3i) = (3 - 2 - 5i)(1 + 3i) =$
$= (1 - 5i)(1 + 3i) = 1 + 3i - 5i - 15i^2 = 1 + 15 - 2i = 16 - 2i$

f) $\frac{2 + 4i}{4 - 2i} = \frac{(2 + 4i)(4 + 2i)}{(4 - 2i)(4 + 2i)} = \frac{8 + 4i + 16i + 8i^2}{16 - 4i^2} = \frac{20i}{16 + 4} = \frac{20i}{20} = i$

g) $\frac{1 - 4i}{3 + i} = \frac{(1 - 4i)(3 - i)}{(3 + i)(3 - i)} = \frac{3 - i - 12i + 4i^2}{9 - i^2} = \frac{3 - 13i - 4}{9 + 1} = \frac{-1 - 13i}{10} =$
$= \frac{-1}{10} - \frac{13}{10}i$

h) $\dfrac{4+4i}{-3+5i} = \dfrac{(4+4i)(-3-5i)}{(-3+5i)(-3-5i)} = \dfrac{-12-20i-12i-20i^2}{9-25i^2} = \dfrac{-12-32i+20}{9+25} =$

$= \dfrac{8-32i}{34} = \dfrac{8}{34} - \dfrac{32}{34}i = \dfrac{4}{17} - \dfrac{16}{17}i$

i) $\dfrac{5+i}{-2-i} = \dfrac{(5+i)(-2+i)}{(-2-i)(-2+i)} = \dfrac{-10+5i-2i+i^2}{4+1} = \dfrac{-10+3i-1}{5} = \dfrac{-11+3i}{5} =$

$= \dfrac{-11}{5} + \dfrac{3}{5}i$

j) $\dfrac{1+5i}{3+4i} = \dfrac{(1+5i)(3-4i)}{(3+4i)(3-4i)} = \dfrac{3-4i+15i-20i^2}{9-16i^2} = \dfrac{3+11i+20}{9+16} =$

$= \dfrac{23+11i}{25} = \dfrac{23}{25} + \dfrac{11}{25}i$

k) $\dfrac{4-2i}{i} = \dfrac{(4-2i)(-i)}{i(-i)} = \dfrac{-4i+2i^2}{1} = -4i-2 = -2-4i$

l) $6-3\left(5+\dfrac{2}{5}i\right) = 6-15+\dfrac{6}{5}i = -9+\dfrac{6}{5}i$

m) $\dfrac{(-3i)^2(1-2i)}{(2+2i)} = \dfrac{9i^2(1-2i)}{(2+2i)} = \dfrac{-9(1-2i)}{(2+2i)} = \dfrac{-9+18i}{(2+2i)} =$

$= \dfrac{(-9+18i)(2-2i)}{(2+2i)(2-2i)} = \dfrac{-18+18i+36i-36i^2}{4-4i^2} = \dfrac{-18+54i+36}{4+4} =$

$= \dfrac{18+54i}{8} = \dfrac{18}{8} + \dfrac{54}{8}i = \dfrac{9}{4} + \dfrac{27}{4}i$

3.

¿Cuánto debe valer x, real, para que $(25-xi)^2$ sea imaginario puro?

$$(25-xi)^2 = 625 + x^2i^2 - 50xi = (625 - x^2) - 50xi$$

Para que sea imaginario puro:

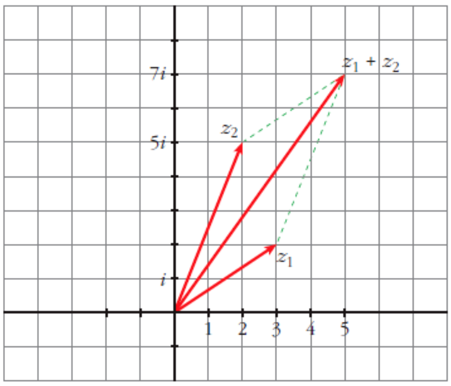
$$625 - x^2 = 0 \;\rightarrow\; x^2 = 625 \;\rightarrow\; x = \pm\sqrt{625} = \pm 25$$

Hay dos soluciones: $x_1 = -25$, $x_2 = 25$

4.

Representa gráficamente $z_1 = 3+2i$, $z_2 = 2+5i$, z_1+z_2. Comprueba que z_1+z_2 es una diagonal del paralelogramo de lados z_1 y z_2.

$z_1 + z_2 = 5 + 7i$



5.

Escribe en forma polar los siguientes números complejos:

a) $1 + \sqrt{3}\,i$ **b)** $\sqrt{3} + i$ **c)** $-1 + i$

d) $5 - 12i$ **e)** $3i$ **f)** -5

a) $1 + \sqrt{3}\,i = 2_{60°}$ b) $\sqrt{3} + i = 2_{30°}$ c) $-1 + i = \sqrt{2}_{135°}$

d) $5 - 12i = 13_{292°\ 37'}$ e) $3i = 3_{90°}$ f) $-5 = 5$

6.

Escribe en forma binómica los siguientes números complejos:

a) $5_{(\pi/6)\ rad}$ **b)** $2_{135°}$ **c)** $2_{495°}$

d) $3_{240°}$ **e)** $5_{180°}$ **f)** $4_{90°}$

a) $5_{(\pi/6)} = 5\left(\cos\frac{\pi}{6} + i\,sen\,\frac{\pi}{6}\right) = 5\left(\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right) = \frac{5\sqrt{3}}{2} + \frac{5}{2}i$

b) $2_{135°} = 2(\cos 135° + i\,sen\,135°) = 2\left(-\frac{\sqrt{2}}{2} + i\,\frac{\sqrt{2}}{2}\right) = -\sqrt{2} + \sqrt{2}i$

c) $2_{495°} = 2_{135°} = -\sqrt{2} + \sqrt{2}i$

d) $3_{240°} = 3(\cos 240° + i\,sen\,240°) = 3\left(-\frac{1}{2} - i\,\frac{\sqrt{3}}{2}\right) = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$

e) $5_{180°} = -5$

f) $4_{90°} = 4i$

7.

Sean los números complejos $z_1 = 4_{60°}$ y $z_2 = 3_{210°}$.

a) Expresa z_1 y z_2 en forma binómica.

b) Halla $z_1 \cdot z_2$ y z_2/z_1, y pasa los resultados a forma polar.

c) Compara los módulos y los argumentos de $z_1 \cdot z_2$ y z_2/z_1 con los de z_1 y z_2 e intenta encontrar relaciones entre ellos.

a) $z_1 = 4_{60°} = 4(\cos 60° + i\,sen\,60°) = 4\left(\frac{1}{2} + i\,\frac{\sqrt{3}}{2}\right) = 2 + 2\sqrt{3}\,i$

$z_2 = 3_{210°} = 3(\cos 210° + i\,sen\,210°) = 3\left(-\frac{\sqrt{3}}{2} - i\,\frac{1}{2}\right) = -\frac{3\sqrt{3}}{2} - \frac{3}{2}i$

b) $z_1 \cdot z_2 = \left(2 + 2\sqrt{3}\,i\right)\left(-\frac{3\sqrt{3}}{2} - \frac{3}{2}i\right) =$

$= -3\sqrt{3} - 3i - 9i - 3\sqrt{3}\,i^2 = -3\sqrt{3} - 12i + 3\sqrt{3} = -12i = 12_{270°}$

$$\frac{z_2}{z_1} = \frac{\left(-\frac{3\sqrt{3}}{2} - \frac{3}{2}i\right)}{\left(2 + 2\sqrt{3}i\right)} = \frac{\left(-\frac{3\sqrt{3}}{2} - \frac{3}{2}i\right)\left(2 - 2\sqrt{3}i\right)}{\left(2 + 2\sqrt{3}i\right)\left(2 - 2\sqrt{3}i\right)} =$$

$$= \frac{-3\sqrt{3} - 3i + 9i + 3\sqrt{3}i^2}{4 - 12i^2} = \frac{-3\sqrt{3} + 6i - 3\sqrt{3}}{4 + 12} = \frac{-6\sqrt{3} + 6i}{16} = \left(\frac{3}{4}\right)_{150°}$$

c) $z_1 \cdot z_2 = 4_{60°} \cdot 3_{210°} = (4 \cdot 3)_{60° + 210°} = 12_{270°}$

$$\frac{z_2}{z_1} = \frac{3_{210°}}{4_{60°}} = \left(\frac{3}{4}\right)_{210° - 60°} = \left(\frac{3}{4}\right)_{150°}$$

8.

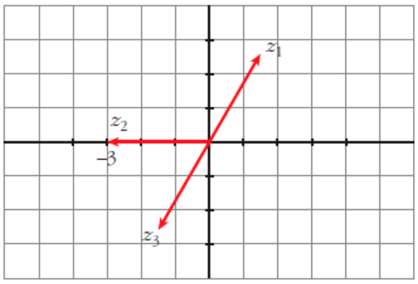
Resuelve la ecuación $z^3 + 27 = 0$. Representa sus soluciones.

$z^3 + 27 = 0 \rightarrow z = \sqrt[3]{-27} = \sqrt[3]{27_{180°}} = 3_{(180° + 360°\, n)/3} = 3_{60° + 120°\, n}$; $n = 0, 1, 2$

$z_1 = 3_{60°} = 3(\cos 60° + i \operatorname{sen} 60°) = 3\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \frac{3}{2} + \frac{3\sqrt{3}}{2}i$

$z_2 = 3_{180°} = -3$

$z_3 = 3_{240°} = 3(\cos 240° + i \operatorname{sen} 240°) = 3\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$



9.

Resuelve las ecuaciones:

a) $z^4 + 1 = 0$

b) $z^6 + 64 = 0$

a) $z^4 + 1 = 0 \rightarrow z = \sqrt[4]{-1} = \sqrt[4]{1_{180°}} = 1_{(180° + 360°\, k)/2} = 1_{45° + 90°\, k}$; $k = 0, 1, 2, 3$

Las cuatro raíces son:

$1_{45°} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$; $1_{135°} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$; $1_{225°} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$; $1_{315°} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

b) $z^6 + 64 = 0 \rightarrow z = \sqrt[6]{-64} = \sqrt[6]{64_{180°}} = 2_{(180° + 360°\, k)/6} = 2_{30° + 60°\, k}$; $k = 0, 1, 2, 3, 4, 5$

Las seis raíces son:

$2_{30°} = 2\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \sqrt{3} + 1$

$2_{90°} = 2i$

$2_{150°} = 2\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = -\sqrt{3} + i$

$2_{210°} = 2\left(-\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) = -\sqrt{3} - i$

$2_{270°} = -2i$

$2_{330°} = 2\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) = \sqrt{3} - i$

10.

Calcula en forma binómica:

a) $\dfrac{(3+3i)(4-2i)}{2-2i}$ **b)** $\dfrac{-2+3i}{(4+2i)(-1+i)}$

c) $\dfrac{2+5i}{3-2i}(1-i)$ **d)** $\dfrac{1+i}{2-i}+\dfrac{-3-2i}{1+3i}$

a) $\dfrac{(3+3i)(4-2i)}{2-2i} = \dfrac{12-6i+12i-6i^2}{2-2i} = \dfrac{18+6i}{2-2i} = \dfrac{(18+6i)(2+2i)}{(2-2i)(2+2i)} =$

$= \dfrac{36+36i+12i-12}{4+4} = \dfrac{24+48i}{8} = 3+6i$

b) $\dfrac{-2+3i}{(4+2i)(-1+i)} = \dfrac{-2+3i}{-4+4i-2i-2} = \dfrac{-2+3i}{-6+2i} = \dfrac{(-2+3i)(-6-2i)}{(-6+2i)(-6-2i)} =$

$= \dfrac{12+4i-18i+6}{36+4} = \dfrac{18-14i}{40} = \dfrac{9-7i}{20} = \dfrac{9}{20} - \dfrac{7}{20}i$

c) $\dfrac{2+5i}{3-2i}(1-i) = \dfrac{2-2i+5i+5}{3-2i} = \dfrac{7+3i}{3-2i} = \dfrac{(7+3i)(3+2i)}{(3-2i)(3+2i)} =$

$= \dfrac{21+14i+9i-6}{9+4} = \dfrac{15+23i}{13} = \dfrac{15}{13} + \dfrac{23}{13}i$

d) $\dfrac{1+i}{2-i} + \dfrac{-3-2i}{1+3i} = \dfrac{(1+i)(2+i)}{(2-i)(2+i)} + \dfrac{(-3-2i)(1-3i)}{(1+3i)(1-3i)} =$

$= \dfrac{2+i+2i-1}{4+1} + \dfrac{-3+9i-2i-6}{1+9} = \dfrac{1+3i}{5} + \dfrac{-9+7i}{10} =$

$= \dfrac{2+6i-9+7i}{10} = \dfrac{-7+13i}{10} = \dfrac{-7}{10} + \dfrac{13}{10}i$